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Estimation of the thermal conductivity of composites

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In this article we introduce the concept of *homogenization* for the approximation of the effective thermal conductivity of composites. A simple algebraic approximation method is proposed and shown to yield an upper bound for the effective conductivity. Numerical results are given for uni-directional carbon-carbon composites which demonstrate the validity of the approach.

1. Introduction

The ability to accurately predict material constants for composite materials has been, and continues to be, an active area of research. One such constant of interest is the (effective) thermal conductivity of the composite. The simplest case for consideration is that of a unidirectional, fiber reinforced, composite in which all the fibers are assumed to be parallel, and periodically distributed throughout the composite. For the estimation of the conductivity parallel to the fiber axis, a simple weighted average of the constituents has been found to yield accurate results [4, 10, 14, 16], as the highly conductive fibers transport most of the heat flow through the composite.

The estimation of the conductivity perpendicular to the fiber axis has not been nearly as successful. In this case the matrix itself is largely responsible for the heat transport, with the fibers, pores and cracks, serving to disrupt the direct heat flow through the composite. Three approaches which have been used are:

(i) the reciprocal of the average of the reciprocal of the constituents (see (4), [6, 14]),
(ii) a variable dispersive model [13, 16],
(iii) estimations based upon calculations for a *fundamental unit* [3, 9, 11].

The effect of porosity(voids and cracks within the composite) is much more pronounced on the conductivity perpendicular to the fiber axis than in the parallel case. In the variable dispersive model a shape parameter is introduced which models the voids in the composite. In the case of a composite composed of isotropic fibers and an isotropic matrix, the variable dispersive model does reasonable well at predicting the conductivity, as the model is based on isotropic material properties. However the model is not as reliable in the case of composites with anisotropic fibers and matrix. The estimates of conductivity described in [11] are obtained via a finite element method approximation of the steady-state heat flow across a "fundamental unit". The approximation procedure described within is much simplier and the estimates obtained are comparable in accuracy to those predicted by [11]. The approach described in [3] relies on the solution of an auxilary problem in an unbounded domain and does not account for the occurance of voids in the composite.

In this article we introduce the concept of *homogenization* for the estimation of conductivities for composites, which falls into the class (iii) described above (Section 2.1). We then propose a simple algebraic approach, based upon considering the composite as being made up of "layers", for the estimation of the effective thermal conductivities (Section 2.2). We show that the proposed method (theoretically) yields an upper bound for the conductivity. A comparison of the theoretical results with measured data is given (Section 3).

2. Modeling thermal conductivity of composites

In order to model the *effective* thermal conductivity of a *composite* we must assume some underlying structure

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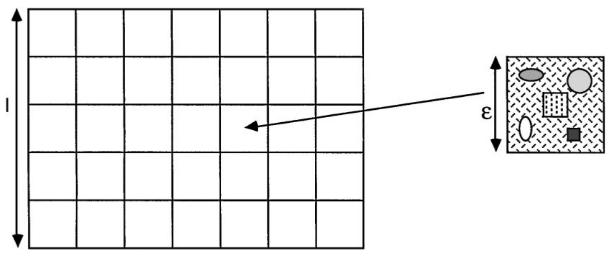

Figure 1 Illustration of a 2D composite.

for the composite. Throughout we will assume that the composite is periodic in all directions and we refer to the unit which is periodically repeated as the **fundamental component** for the composite. (See Fig. 1 for a 2D illustration.) The effective thermal conductivity is a "bulk" property of the *composite* and is independent of the 'size' of the composite. It corresponds to considering the material to be macroscopically homogeneous. Mathematically this corresponds to considering the values associated with the composite as the ration of ϵ/l goes to zero. (See [2, 15] for a precise description of this limiting process.) Following a description of the notation used, in Section 2.1 we give the characterization of the effective thermal conductivity matrix for a composite from homogenization theory. Presented in Section 2.2 is a simplified model which yields an upper bound to the values of the effective thermal conductivity matrix by purely algebraic means. A comparison of the models for two **fundamental components** is given.

Notation

Following standard notation, we denote by $L^2(Q)$ the space of functions defined on Q which are square integrable, and $H^1(Q)$ functions, which along with themselves being square integrable, having a square integrable gradient (see [12]). The norm associated with $H^1(Q)$ is

$$\|u\|_{H^1}^2 := \|u\|_0^2 + |u|_1^2,$$

where

$$\|u\|_0^2 := \int_Q u^2 \, \mathrm{d}x, \quad |u|_1^2 := \int_Q |\nabla u|^2 \, \mathrm{d}x.$$

The space $H_0^1(Q)$ denotes those functions in $H^1(Q)$ which are zero on ∂Q, the boundary of Q.

Consider functions defined in $\mathbb{R}^m$ and periodic in each argument $x_1, x_2, \ldots, x_m$ with periods $l_1, l_2, \ldots, l_m$, respectively. Let $\blacksquare$ represent the basic parallelepiped whose edges are directed along the co-ordinate axes and have respective lengths $l_1, l_2, \ldots, l_m$. By $\langle g \rangle$ we represent the mean value of the periodic function $g(x)$, i.e.,

$$\langle g \rangle = \frac{1}{|\blacksquare|} \int_{\blacksquare} g(x) \, \mathrm{d}x,$$

where $|\blacksquare|$ denotes the volume of the parallelpiped $\blacksquare$. The space of H^1 functions with period $\blacksquare$ we denote by $H^1(\blacksquare)$.

The notation $H^{-1}(Q)$ denotes the "dual space" of $H^1(Q)$, i.e. the set of all continuous linear functionals on $H_0^1(Q)$, (see[12]). If f is an element of $H^{-1}(\blacksquare)$ then (f, ϕ) represents the value of the functional applied to $\phi \in H_0^1(Q)$.

Strong convergence in $H_0^1(Q)$ is denoted by the symbol $\to$. Strong convergence refers to convergence with respect to the norm, i.e. $u^\epsilon \to u^0$ if $\lim_{\epsilon\to 0} \|u^\epsilon - u^0\|_{H_0^1(Q)} = 0$. *Weak convergence* is denoted by $\rightharpoonup$. We write $u^\epsilon \rightharpoonup u^0$ in $H_0^1(Q)$ if $\lim_{\epsilon\to 0}(f, u^\epsilon) = (f, u^0)$ for all $f \in H^{-1}(Q)$. Weak convergence may be interpreted in the indirect sense of "action", i.e. if the action of f on u^ϵ converges to the action of f on u^0 as $\epsilon \to 0$ for all $f \in H^{-1}(Q)$, then $u^\epsilon \rightharpoonup u^0$.

Convergence in $L^2(Q)$ is defined analogously.

2.1. Homogenization model

In this section we follow the notation and presentation in [15]. Let $\mathcal{K}(x)$, $x \in \mathbb{R}^m$ be a periodic matrix with bounded elements, k_{ij}, satisfying the ellipticity condition

$$\sum_{ij} k_{ij}\eta_i\eta_j \geq \nu_1|\eta|, \quad \text{for all } \eta, x \in \mathbb{R}^m, \text{ where } \nu_1 > 0. \tag{1}$$

We assume that the matrix

$$\mathcal{K}^\epsilon(x) = \mathcal{K}(\epsilon^{-1}x)$$

characterizes a *micro-nonhomogeneous medium*.

Definition 1 ([15], p. 12). A constant positive definite matrix $\mathcal{K}^0$ is said to be the homogenized matrix for $\mathcal{K}(x)$, if for any bounded domain $Q \subset \mathbb{R}^m$ and any $f \in H^{-1}(Q)$ the solutions u^ϵ of the Dirichlet problem

$$u^\epsilon \in H_0^1(Q), \quad \mathrm{div}(\mathcal{K}^\epsilon \nabla u^\epsilon) = f,$$

possess the following convergence properties:

$$u^\epsilon \xrightharpoonup{H_0^1(Q)} u^0, \quad \mathcal{K}^\epsilon \nabla u^\epsilon \xrightharpoonup{L^2(Q)} \mathcal{K}^0 \nabla u^0,$$

as $\epsilon \to 0$, where u^0 is the solution of the Dirichlet problem

$$u^0 \in H_0^1(Q), \quad \mathrm{div}(\mathcal{K}^0 \nabla u^0) = f.$$

Theorem 1 ([15], p. 18). *Let $\mathcal{K}$ be a symmetric periodic matrix with bounded elements satisfying the ellipticity condition* (1). *Then the symmetric matrix $\mathcal{K}^0$ defined by*

$$\lambda \cdot \mathcal{K}^0 \lambda = \inf_{v \in H^1(\blacksquare)} \langle (\lambda + \nabla v) \cdot \mathcal{K}(\lambda + \nabla v) \rangle, \tag{2}$$

is the homogenized matrix for $\mathcal{K}$ in the sense of Definition 1.

2.2. Layered approximating model

The model we present in this section is based on approximating the composite by a *layered* composite. We

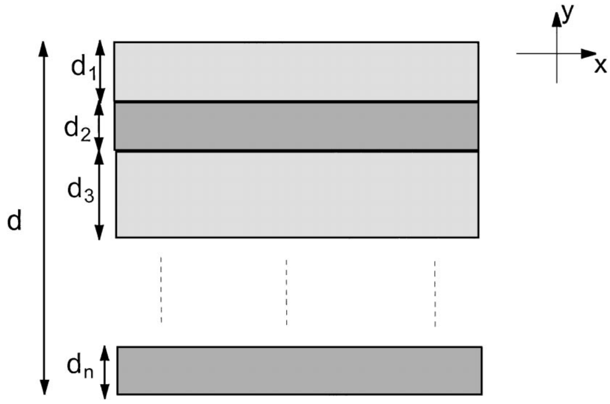

Figure 2 Homogeneous anistropic layered material.

restrict our attention to composites whose components share the same principal axes of heat conduction. For clarity of exposition our presentation is in 2D. We begin by giving a simple derivation for the effective thermal conductivity for a layered composite. The derivation is analogous to the presentation in [1, 8] for the effective hydraulic conductivity in a layered medium in groundwater flow.

Consider the *layered* composite illustrated in Fig. 2. Within each of the layers, l_i having width d_i, we assume that the material is homogeneous and anisotropic with, the principal axes of thermal conductivity aligned with the x–y axes with conductivity values k_i^x, and k_i^y, respectively.

Based on these assumptions we have the following theorem.

Theorem 2. *For a 2D 'vertically' layered composite, whose layers have the same principal axes of heat conduction, the effective thermal conductivies of the composite* k_{eff}^x, *and* k_{eff}^y, *are given by*

$$k_{\text{eff}}^x = \sum_{i=1}^{n} \frac{d_i}{d} k_i^x, \tag{3}$$

and

$$k_{\text{eff}}^y = \frac{d}{\sum_{i=1}^{n} \left(d_i / k_i^y\right)}, \tag{4}$$

where d_i *denotes the height of layer* l_i, *having thermal conductivities* k_i^x, *and* k_i^y, *and* $d = \sum_{i=1}^{n} d_i$.

Proof: To establish (3), we assume the model composite extends periodically in the y-direction and a constant temperature difference, ΔT, is maintained across the entire composite in the x-direction. Denoting by q_i^x the heat-flux at the left of the material along layer l_i we have the amount of heat (per unit time) flowing through l_i is given by $q_i^x d_i$. Using the basic constitutive relation between heat-flux, temperature gradient, and thermal conductivity we have $q_i^x = k_i^x \nabla T^x$. Thus, the total heat (per unit time) entering the composite, H, is given by

$$H = \sum_{i=1}^{n} k_i^x d_i \nabla T^x. \tag{5}$$

Now, treating the composite as a single component with x-directional thermal conductivity k_{eff}^x, we have that $q_{\text{eff}}^x = k_{\text{eff}}^x \nabla T^x$, and

$$H = k_{\text{eff}}^x d \nabla T^x. \tag{6}$$

Equating (5), (6), yields (3).

For the case of heat flow perpendicular to the layers, which are assumed to be of infinite extent in the x-direction, we assume a constant temperature gradient ∇T^y is maintained across the composite in the y-direction. From the conservation of heat, we have that the amount of heat (per unit time, per unit length in the x-direction, $\tilde{H}$, is constant flowing through each of the layers. Thus,

$$\tilde{H} = k_1^y \nabla T_1^y = k_2^y \nabla T_2^y = \cdots = k_n^y \nabla T_n^y, \tag{7}$$

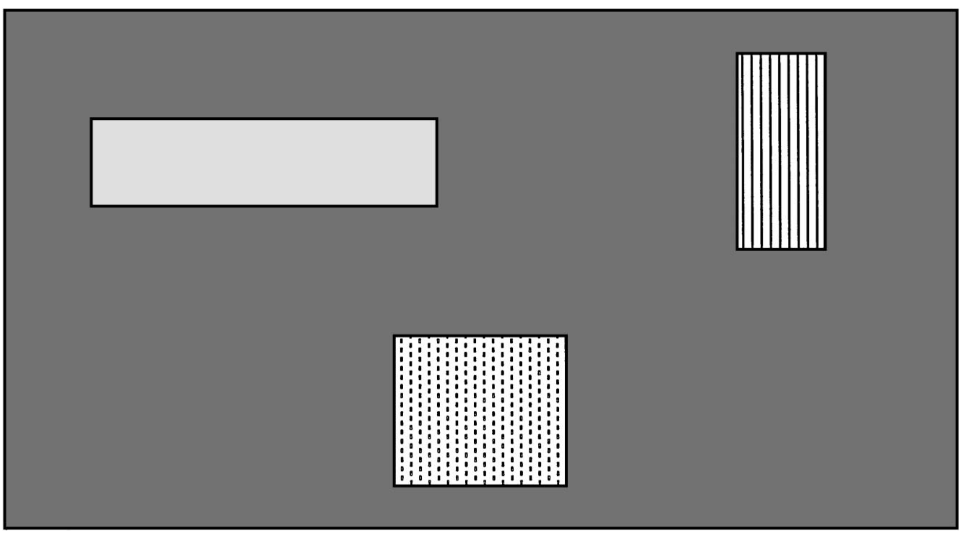

Figure 3 Example of a 2D composite of parallelepiped type.

where ∇T_i^y denotes the temperature gradient across l_i in the y-direction. Treating the composite as a single material with a temperature gradient ∇T^y and thermal conductivity k_{eff}^y we have

$$\tilde{H} = k_{\text{eff}}^y \nabla T^y. \tag{8}$$

Combining (7) and (8) together with

$$\nabla T^y = \sum_{i=1}^{n} \nabla T_i^y,$$

we obtain (4). □

Note: Equations 3, and 4 also follow from 2, see [15], p. 15.

We next introduce a class of composites which we refer to as *parallelepiped type.*

Definition 2 (Composite of parallelepiped type). A composite in $\mathbb{R}^n$ is said to be of parallelepiped type if its **fundamental component** consists of an underlying material M (matrix) and disjoint constitutive components $\mathcal{C}_i$, $i = 1, \ldots, N$, where the $\mathcal{C}_i$ are parallelepipeds.

In $\mathbb{R}^2$ a composite of parallelepiped type is made up of an underlying matrix with disjoint rectangular constitutive components (see Fig. 3).

For a composite of parallelepiped type occupying $[a, b] \times [c, d]$, let $l_\alpha^V := [a_\alpha, b_\alpha] \times [c, d]$, $\alpha = 1, \ldots, \eta$ denote a partitioning of the **fundamental component** into vertical strips, in which the thermal conductivities are only functions of height, y (see Fig. 4). Using (3) and (4), the effective x- and y-thermal conductivities within each layer l_α^V can be calculated, which we denote by k_α^{Vx} and k_α^{Vy}, respectively.

Correspondingly, consider the **fundamental component** partitioned into horizontal layers $l_\beta^H := [a, b] \times [c_\beta, d_\beta]$, $\beta = 1, \ldots, \mu$, in which the thermal conductivities are only functions of the horizontal variable x. Again, using (3) and (4), the effective thermal conductivities within each layer l_β^H may be calculated, which we denote by k_β^{Hx} and k_β^{Hy}.

Now, consider two "layered" composites, C^V and C^H, where C^V has vertical layers of widths $(b_\alpha - a_\alpha)$, $\alpha = 1, \ldots, \eta$ with conductivites k_α^{Vx} and k_α^{Vy}, and C^H has horizontal layers with heights $(d_\beta - c_\beta)$, $\beta = 1, \ldots, \mu$ with conductivites k_β^{Hx} and k_β^{Hy}. Again, using (3) and (4), we can compute the effective thermal conductivity matrices for C^V and C^H, which we denote respectively as

$$\mathcal{K}_{\text{eff}}^{C^V} = \begin{pmatrix} k_{\text{eff}}^{C^{Vx}} & 0 \\ 0 & k_{\text{eff}}^{C^{Vy}} \end{pmatrix} \quad \text{and} \quad \mathcal{K}_{\text{eff}}^{C^H} = \begin{pmatrix} k_{\text{eff}}^{C^{Hx}} & 0 \\ 0 & k_{\text{eff}}^{C^{Hy}} \end{pmatrix}. \tag{9}$$

Both matrices $\mathcal{K}_{\text{eff}}^{C^V}$ and $\mathcal{K}_{\text{eff}}^{C^H}$ are approximations to the effective thermal conductivity matrix for the composite of parallelepiped type. In fact,

Theorem 3. *For a composite of parallelelepiped type the (homogenized) effective thermal conductivity satisfies*

$$k_{11}^0 \le k_{\text{eff}}^{C^{Vx}}, \tag{10}$$

$$k_{22}^0 \le k_{\text{eff}}^{C^{Hy}}. \tag{11}$$

Proof: We establish (10), with (11) following by an analogous argument. To show that $k_{11}^0 \le k_{\text{eff}}^{C^{Vx}}$ we begin with (2).

$$\begin{aligned}
k_{11}^0 &= \inf_{u \in H^1(\blacksquare)} \langle (\lambda + \nabla u) \cdot \mathcal{K}(\lambda + \nabla u) \rangle \text{ with } \lambda = [1, 0]^T \\
&\le \inf\{\langle (\lambda + \nabla u) \cdot \mathcal{K}(\lambda + \nabla u) \rangle, \quad \lambda = [1, 0]^T, \quad u(x, y) = f(x) \in H^1(\blacksquare)\} \\
&= \inf\left\{ \frac{1}{(b-a)(d-c)} \sum_{\alpha=1}^{n} \int_{l_\alpha^V} \int (1 + f'(x)) k_{11}(x, y)(1 + f'(x))\, \mathrm{d}y\, \mathrm{d}x, \quad u(x, y) = f(x) \in H^1(\blacksquare) \right\} \\
&= \inf\left\{ \frac{1}{(b-a)(d-c)} \sum_{\alpha=1}^{\eta} \int_{a_\alpha}^{b_\alpha} (1 + f'(x))^2 \int_c^d k_{11}(x, y)\, \mathrm{d}y\, \mathrm{d}x, \quad u(x, y) = f(x) \in H^1(\blacksquare) \right\} \\
&= \inf\left\{ \frac{1}{(b-a)(d-c)} \sum_{\alpha=1}^{\eta} \int_{a_\alpha}^{b_\alpha} (1 + f'(x))(d - c) k_\alpha^{Vx} (1 + f'(x))\, \mathrm{d}x, \quad u(x, y) = f(x) \in H^1(\blacksquare) \right\} \\
&= \inf\left\{ \frac{1}{(b-a)} \sum_{\alpha=1}^{\eta} \int_{a_\alpha}^{b_\alpha} (1 + f'(x)) k_\alpha^{Vx} (1 + f'(x))\, \mathrm{d}x, u(x, y) = f(x) \in H^1(\blacksquare) \right\} && (12) \\
&= k_{\text{eff}}^{C^{Vx}}, && (13)
\end{aligned}$$

as (12) represents the effective thermal x-conductivity of C^V. □

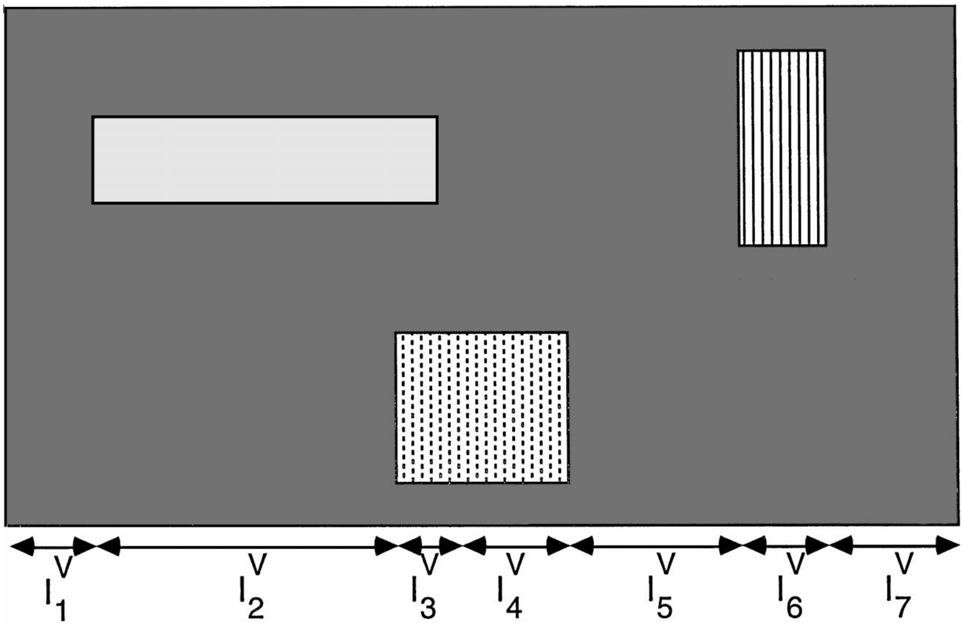

Figure 4 Partition of composite into vertical *layers*.

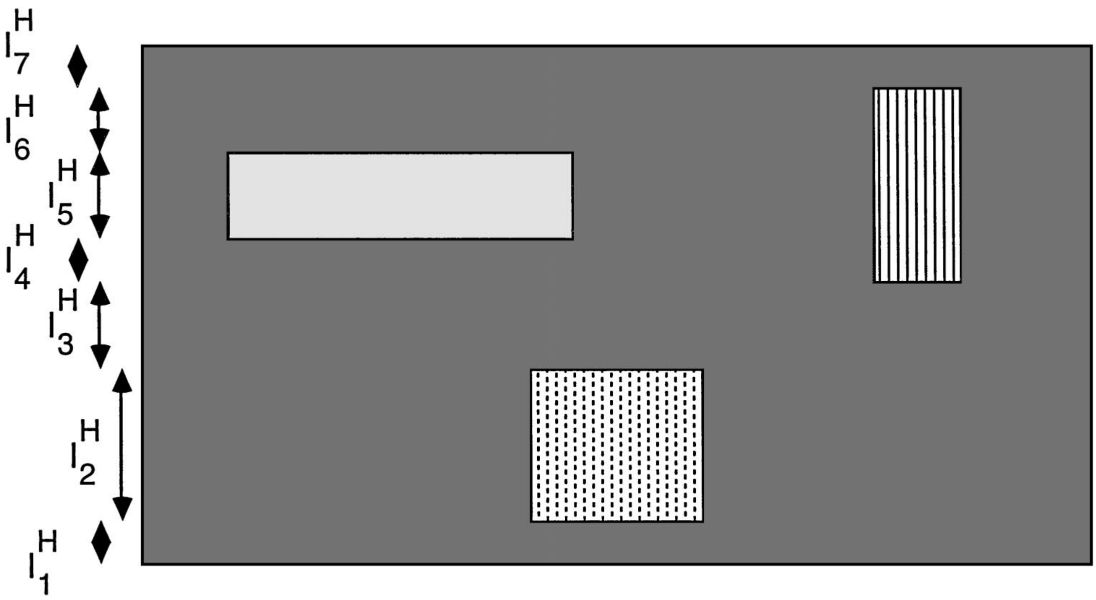

Figure 5 Partition of composite into horizontal *layers*.

The attractiveness of the approximation described in this section is its algebraic simplicity. Using the homogenization theory described in Section 2.1 an approximation to the effective thermal conductivity matrix requires one to formulate and solve a numerical approximation to (2), see [7]. The approximation approach presented in this section enables closed form approximations for the thermal conductivities to be determined. These closed form expressions may be used in determining if particular features of a composite play a significant role in determining the effective thermal conductivities. Below in Table I we compare the homogenized value for the conductivities for two composites, illustrated in Fig. 6 with their layered approximations. The fundamental units A and B were chosen to model a composite comprised of a matrix with a centered (square) fiber and 4 pores. Two of the pores, positioned beside the fiber, model cracks at the fiber matrix interface, and the other pores represent cracks within the matrix itself. The volumes of matrix, fiber, and pores (assumed isotropic) used were (see Sample 19, Table III) 22.5 , 48.6 , 28.9%. The value used for the conductivity of the pores was $k_v = 2.4 \times 10^{-2}$ W/mK which represents the conductivity of air at room temperature.

TABLE I Homogenized and layered approx. of effective k_x (W/m K)

	$k_m = 64.3$ $k_f = 0.76$		$k_m = 31.42$ $k_f = 4.30$	
	Homog.	Layered	Homog.	Layered
Unit A	7.78	8.72	5.10	7.14
Unit B	8.05	9.26	5.04	7.23

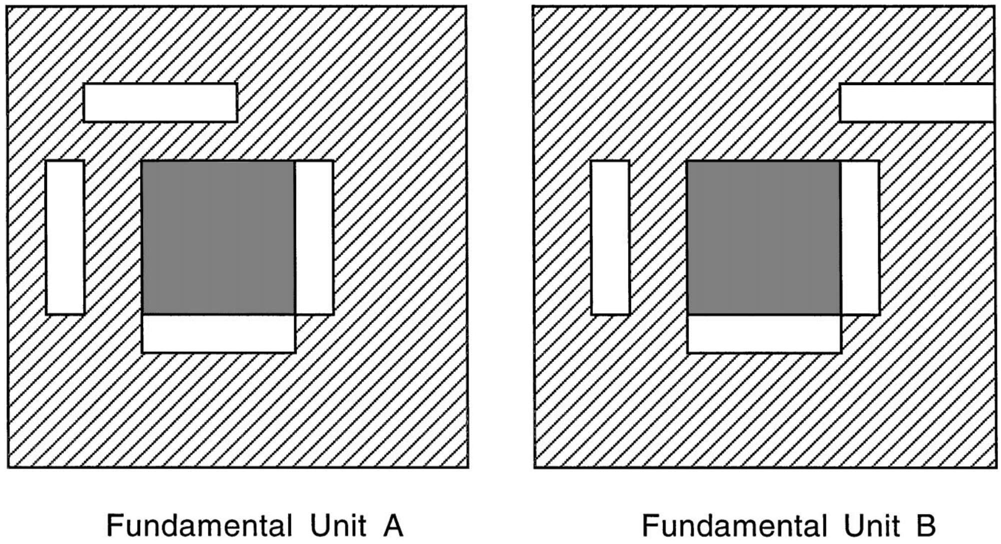

Figure 6 Two fundamental units for a 2D composite.

3. Estimation of thermal conductivities

In this section we demonstrate the effectiveness of the layered approximation approach described in Section 2.2 to the estimation of the thermal conductivities of carbon-carbon composites.

3.1. Carbon-carbon composites

To test the model, two composite types made by Klett [11] were used as baseline composites. These composites were made by a towpregging process whereby a Mitsubishi AR mesophase pitch (designated AR-120) was used as the matrix around $T300$ and $P55$ carbon fibers. The thermal diffusivity of these composites was measured at room temperature on a Xenon Pulse Flash measurement device at Oak Ridge National Laboratory. The thermal conductivity was then calculated using material properties. For each sample composite variables, such as volume fraction of fibers, volume fraction of voids, etc., were determined by optical image analysis.

3.1.1. Parallel to the fibers

Presented in Table II is the data describing the collection of carbon-carbon composites from [11], including the measured thermal conductivities in the direction parallel to the fibers. To obtain the algebraic estimates, also presented in Table II and illustrated in Fig. 7, we apply the simple weighted average Equation 3, corresponding to heat flow in a layered material with layers parallel to the direction of heat flow. The values of the conductivities for the matrix, fibers, and voids used were $k_m = 6.2$ W/mK, $k_f = 8.5$ W/mK, and $k_v = 2.4 \times 10^{-2}$ W/mK, respectively.

To determine the value of the matrix conductivity of the composites, Klett [11] measured the thermal conductivities of three samples from each batch of composites. The error associated with the measured value of thermal conductivity may be as high as 5% with a 95% confidence interval. Also required in order to back out the matrix conductivity was the measurement of void fraction which was estimated to within 5%. Thus the error in the value used for the matrix conductivity of the composites is accurate to at best 5%.

TABLE II Thermal conductivity (in W/mk) in the direction parallel to the fibers

Sample	Fiber Vol %	Void Vol %	Matrix Vol %	Measured	Estimated
1	48.5	12.7	38.8	5.9	6.53
2	48.5	12.5	39.0	6.1	6.54
3	34.5	28.2	37.3	4.42	5.25
4	34.5	33.1	32.4	4.79	4.95
5	34.5	34.8	30.7	4.86	4.84
6	34.5	34.8	30.7	4.62	4.84
7	34.5	31.7	33.8	4.47	5.04
8	34.5	29.4	36.1	4.59	5.18
9	34.5	38.3	27.2	4.79	4.63
10	34.5	36.2	29.3	4.74	4.76
11	34.5	33.3	32.2	5.26	4.94
12	34.5	41.2	24.3	4.59	4.45
13	34.5	35.3	30.2	4.71	4.81
14	34.5	37.0	28.5	4.98	4.71
15	34.5	37.8	27.7	5.11	4.66
16	34.5	34.2	31.3	4.71	4.88
17	34.5	38.9	26.6	4.42	4.59
18	34.5	39.1	26.4	4.54	4.58
19	34.5	39.2	26.3	5.0	4.57
20	34.5	41.5	24.0	4.52	4.43

From the data presented in Table II and the associated graph in Fig. 7, the simple weighted average formula, represented by the solid line, does a reasonable job of estimating the conductivities, given the uncertainities in the values for the conductivities of the matrix and the fiber, and that of the measured conductivities.

3.2. Perpendicular to the fibers

In modeling the conductivity perpendicular to the fibers we assume that within the fundamental unit there are four rectangular voids of equal size, with the largest side having length equal to the side length of the fiber, which we model as being square. (Though the square and rectangular geometry is not physically accurate, we show in Section 4 that the difference in thermal

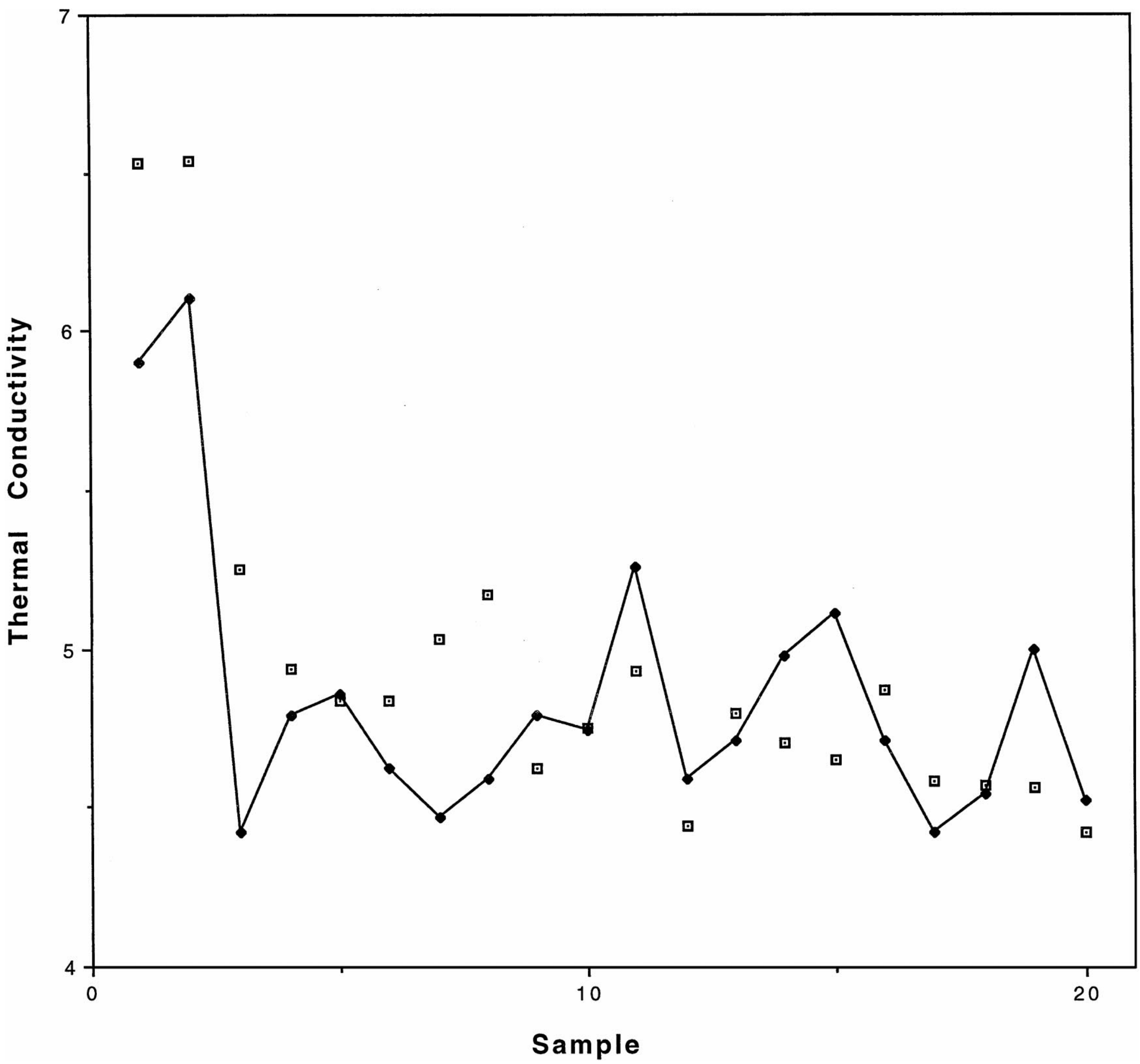

Figure 7 Estimated and measured values of thermal conductivity in the direction parallel to the fibers.

conductivity using a square and circular fiber is negligible.) Given the volume percentages of fibers and voids in each sample, the dimensions of the fiber and voids in the fundamental units, see Fig. 6, are determined. Using the computational procedure described following Definition 2 for approximating the conductivity of a composite of parallelepiped type, an estimate (a guaranteed upper bound, see Theorem 3) for the conductivity of the fundamental unit is computed. For the results given in Table III, and illustrated graphically in Fig. 8, we use both configurations presented in Fig. 6 as "fundamental units" and then averaged the predicted conductivities. The conductivity values used for the matrix and fibers were those given in [11]. These values were derived based on the assumption that *the conductivity perpendicular to the basal plane should be two orders of magnitude less than that parallel to the basal plane.*

The layered estimates appeared to follow the trends of the measured data but overcompensate when the trend would change. To compensate this behavior, using sample values 5 and 12 we applied a non-linear minimization to determine values for k_m and k_f which would best "fit" these two data points. The values obtained were $k_m = 31.23$ W/mK, $k_f = 4.30$ W/mK. Using these values for the conductivities the layered approximation again captures the trends exhibited by the measured values and in addition more closely matches these values.

4. Modeling effect of square versus circular fibers

In keeping with the approach of constructing a "simple" approximation method for the estimation of thermal conductivity we have modeled the fibers as having square cross-section. For heat transport parallel to the fiber axis one obtains the same value for the approximation for fibers with square or circular cross-section (assuming the same cross-sectional areas). The situation for flow perpendicular to the fiber axis is more complicated. In [7] numerical experiments were performed to compare the difference in the effective thermal conductivities of square and circular fibers in the

TABLE III Thermal conductivity W/mK in the direction perpendicular to the fibers

Sample	Fiber Vol %	Void Vol %	Matrix Vol %	Measured	Estimated $k_m = 64.3$ $k_f = 0.76$	Estimated $k_m = 31.23$ $k_f = 4.30$
1	54.3	7.3	38.4	10.20	17.47	11.46
2	57.3	8.6	34.1	16.10	15.12	10.38
3	60.6	11.0	28.4	16.10	12.21	9.01
4	40.7	41.9	17.4	3.84	4.41	5.28
5	40.7	42.9	16.4	4.79	3.64	4.99
6	40.7	44.5	14.8	3.32	2.36	4.53
7	40.7	45.6	13.7	4.12	1.42	4.20
8	40.7	38.8	20.5	9.59	6.64	6.15
9	40.7	44.6	14.7	5.96	2.27	4.50
10	60.6	25.0	14.4	6.82	5.60	5.79
11	60.6	32.3	7.1	5.57	1.98	4.25
12	60.6	25.9	13.5	6.38	5.17	5.61
13	60.6	26.3	13.1	6.66	4.97	5.51
14	60.6	28.0	11.4	6.19	4.15	5.15
15	48.6	26.0	25.4	5.24	10.57	7.90
16	48.6	33.0	18.4	6.50	6.68	6.18
17	48.6	29.9	21.5	6.64	8.43	6.94
18	48.6	31.7	19.7	6.44	7.43	6.50
19	48.6	28.9	22.5	6.11	8.99	7.18
20	48.6	27.2	24.2	7.40	9.92	7.60
21	48.6	29.1	22.3	5.86	8.88	7.14
22	58.4	30.6	11.0	4.45	3.75	4.98
23	58.4	27.0	14.6	6.28	5.56	5.76
24	58.4	24.9	16.7	3.88	6.58	6.22
25	58.4	23.8	17.8	7.35	7.12	6.46
26	58.4	29.6	12.0	6.78	4.26	5.19

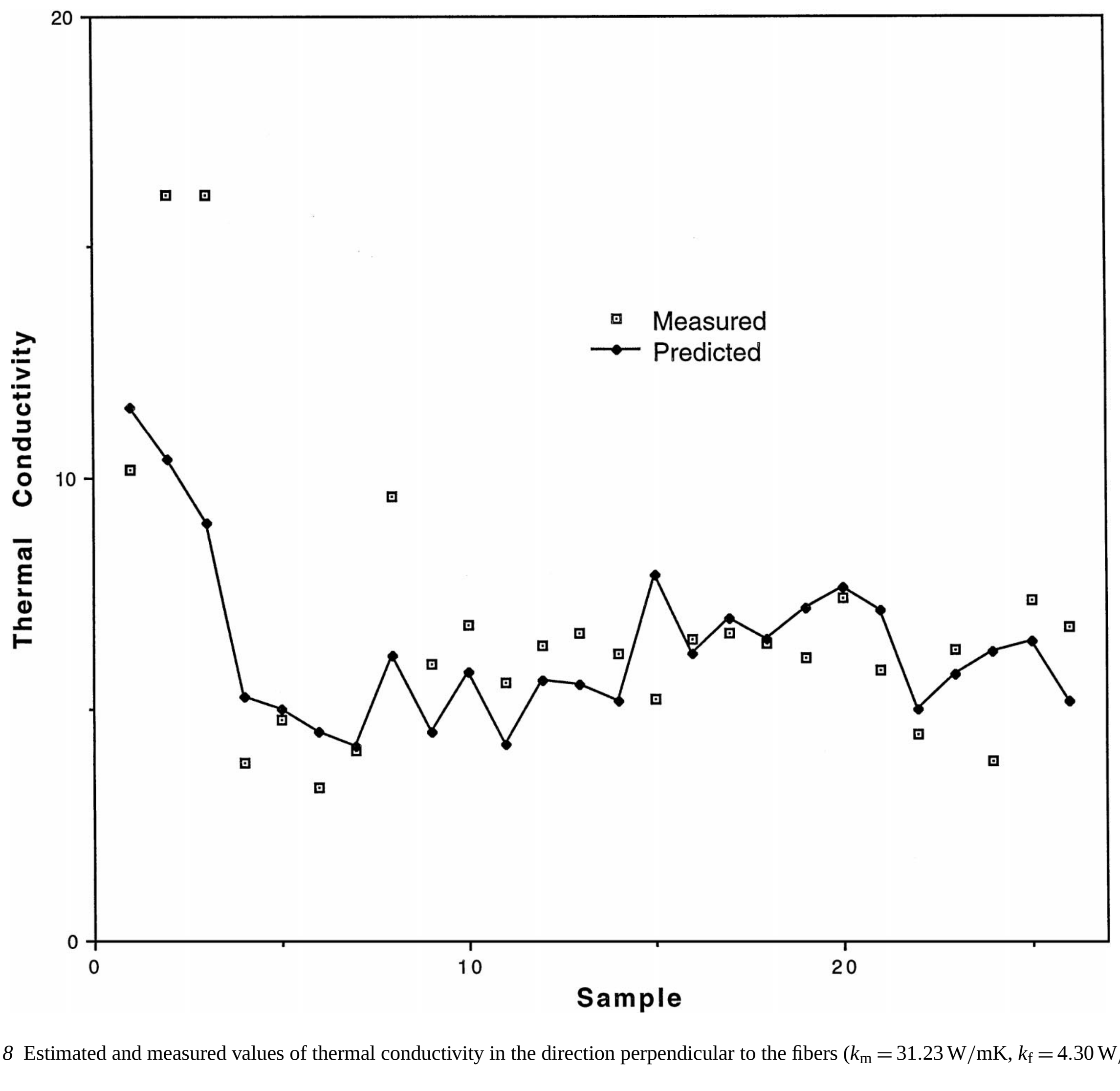

Figure 8 Estimated and measured values of thermal conductivity in the direction perpendicular to the fibers ($k_m = 31.23$ W/mK, $k_f = 4.30$ W/mK).

fundamental unit, for flow perpendicular to the fiber axis. For $k_m = 1$ (fixed), k_f values of 0.10, 0.25, 0.50, and 0.75, and for fiber volumes, $V_f = 25$ and 50% the difference in the conductivities between the composites having square and circular fibers was, in all cases, less than 2%. Hence, modeling using square fibers is clearly justified.

5. Concluding remarks

In this article we demonstrated that a "layered" approximation gives an easily computable, reasonable accurate, approximation for the effective thermal conductivity of carbon-carbon composites. It is interesting to note the accuracy of the estimation in view of the fact that this approach completely ignores the micro-structure of the composites, and only uses very coarse assumptions about the macro-structure. This indicates that the important factors in estimating effective thermal conductivity of composites are the constitutive parameters, such as the volumes of the various components and their thermal properties. Issues such as crack profiles and distributions are of secondary importance.

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